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Report No. CG-D-05-87

**PATROL:**  
**VOLUME I - MODEL DESCRIPTION AND ANALYST'S GUIDE**

**CLARK W. PRITCHETT**

U.S. COAST GUARD RESEARCH AND DEVELOPMENT CENTER  
AVERY POINT, GROTON, CONNECTICUT 06340-6096



**FINAL REPORT  
SEPTEMBER 1986**

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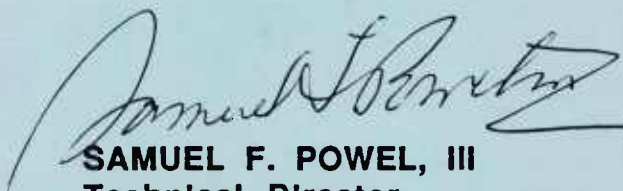
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United States Coast Guard  
Office of Research and Development  
Washington, DC 20593

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# Technical Report Documentation Page

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16. Abstract  A mathematical model of a Coast Guard cutter on a law enforcement patrol is described in this report. The kernel of the model is a Markov process that uses the phases of the patrol, such as search or transit, as states of the system. The phases of the patrol are separated by events, such as a detection which terminates a search and potentially initiates the pursuit of a vessel. A computer program called PATROL implements the model described here. The information that describes the law enforcement patrol is organized into four categories: Cutter, Traffic, Area, and Choices. Model outputs are grouped under three headings: Allocation of Effort, Vessel Performance, and Logistics. Distance, time, and fuel consumption information for every component of the patrol can be printed out at the option of the user. PATROL has a range of potential uses. These include vessel assessment, scenario development, accessing policy, strategy and tactics, and understanding the inter-relationships of the different parts of the patrol problem. The appendices of this report include detailed calculations of the probability of detection and interception algorithms used in the model to calculate the time between detections and the time to intercept. This is one volume of a three volume set that describes PATROL. Volume II is a user's manual for the model. Volume III includes all of the programmer level documentation necessary to maintain the model.			
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# METRIC CONVERSION FACTORS

## Approximate Conversions to Metric Measures

Symbol When You Know Multiply By To Find Symbol

LENGTH				
in	inches	* 2.5	centimeters	cm
ft	feet	30	centimeters	cm
yd	yards	0.9	meters	m
mi	miles	1.6	kilometers	km

AREA				
in <sup>2</sup>	square inches	6.5	square centimeters	cm <sup>2</sup>
ft <sup>2</sup>	square feet	0.09	square meters	m <sup>2</sup>
yd <sup>2</sup>	square yards	0.8	square meters	m <sup>2</sup>
mi <sup>2</sup>	square miles	2.6	square kilometers	km <sup>2</sup>
	acres	0.4	hectares	ha

MASS (WEIGHT)				
oz	ounces	28	grams	g
lb	pounds	0.45	kilograms	kg
	short tons (2000 lb)	0.9	tonnes	t

VOLUME				
tsp	teaspoons	5	milliliters	ml
tbsp	tablespoons	15	milliliters	ml
fl oz	fluid ounces	30	milliliters	ml
c	cups	0.24	liters	l
pt	pints	0.47	liters	l
qt	quarts	0.95	liters	l
gal	gallons	3.8	liters	l
ft <sup>3</sup>	cubic feet	0.03	cubic meters	m <sup>3</sup>
yd <sup>3</sup>	cubic yards	0.76	cubic meters	m <sup>3</sup>

TEMPERATURE (EXACT)				
°F	Fahrenheit temperature	5/9 (after subtracting 32)	Celsius temperature	°C

\* 1 in = 2.54 (exactly). For other exact conversions and more detailed tables, see NBS Misc. Publ. 286, Units of Weights and Measures. Price \$2.25. SD Catalog No. C13.10.286.

## Approximate Conversions from Metric Measures

Symbol When You Know Multiply By To Find Symbol

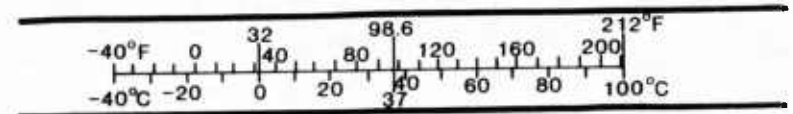
LENGTH				
mm	millimeters	0.04	inches	in
cm	centimeters	0.4	inches	in
m	meters	3.3	feet	ft
m	meters	1.1	yards	yd
km	kilometers	0.6	miles	mi

AREA				
cm <sup>2</sup>	square centimeters	0.16	square inches	in <sup>2</sup>
m <sup>2</sup>	square meters	1.2	square yards	yd <sup>2</sup>
km <sup>2</sup>	square kilometers	0.4	square miles	mi <sup>2</sup>
ha	hectares (10,000 m <sup>2</sup> )	2.5	acres	

MASS (WEIGHT)				
g	grams	0.035	ounces	oz
kg	kilograms	2.2	pounds	lb
t	tonnes (1000 kg)	1.1	short tons	

VOLUME				
ml	milliliters	0.03	fluid ounces	fl oz
l	liters	0.125	cups	c
l	liters	2.1	pints	pt
l	liters	1.06	quarts	qt
l	liters	0.26	gallons	gal
m <sup>3</sup>	cubic meters	35	cubic feet	ft <sup>3</sup>
m <sup>3</sup>	cubic meters	1.3	cubic yards	yd <sup>3</sup>

TEMPERATURE (EXACT)				
°C	Celsius temperature	9/5 (then add 32)	Fahrenheit temperature	°F





U.S. Department  
of Transportation

United States  
Coast Guard



# Memorandum

Subject: SUBMISSION OF R&D REPORT, "PATROL: VOLUME I -  
MODEL DESCRIPTION AND ANALYST'S GUIDE"

Date: MAP 9 1987  
3915 1221R

From: G-D

Reply to: G-DMT-2: 267-1058  
Attn of: LT SMITH

To: DISTRIBUTION

1. Enclosed is a copy of Report Number CG-D-5-87, "PATROL: Volume I - Model Description and Analyst's Guide." This is a final report for R&D Project 9207, Marine Vehicle Technology. This is volume I of III. The other volumes are for official use only and may be obtained by request from the R&D Center.

2. This report describes a model of a Coast Guard cutter or candidate craft on a law enforcement patrol. The inputs to the model have been organized to make it easy for the user to provide the information necessary to run the program. Inputs are required to describe the cutter, area of operation, traffic in the region and choices in the operation of the cutter. Three broad categories of outputs are produced by the model. Allocation of effort shows distances and time spent in each phase of the patrol. Vessel performance presents various measures of effectiveness such as interdiction rate. Logistic information includes mission profiles, fuel consumption information and a detailed breakout of all support tasks on the patrol. Vessel description and scenario input and output data are stored for easy retrieval.

3. The computer program (PATROL) runs on the VAX-750 in the Office of Research and Development at Coast Guard Headquarters and on the micro-VAX II at the R&D Center. It is maintained by the Marine Systems Branch of the R&D Center. This analysis tool was developed to evaluate candidate craft for acquisition in ELT mission performance. It has been successfully used in the WPB candidate craft evaluations and also has great potential to explore variations of ELT tactics and possibly evaluate some military operation applications. To obtain maximum operational value from this model, it should be presented to people in the field so they can have the opportunity to exercise it and provide feedback to the Research and Development Center to improve the model.

4. This report will be available to the public through the National Technical Information Service (NTIS), Springfield, Virginia 22161.

  
D. F. CUNNINGHAM  
Acting

Encl: (1) Report Number CG-D-5-87

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vols 2 & 3  
only  
useful  
if actually  
using  
program*

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## ORGANIZATION

This is one volume of a three volume set that describes PATROL, a model of a Coast Guard cutter in a law enforcement operation. The three volumes (and their classification) are:

PATROL: Volume I - Model Description and Analyst's Guide (UNCLAS.)  
by Clark W. Pritchett  
USCG Research and Development Center  
Groton, CT

PATROL: Volume II - A User's Guide (FOUO)  
by Kelly Frankhouser  
Vitro Corporation  
New London, CT

PATROL: Volume III - Program Description (FOUO)  
by Kelly Frankhouser  
Vitro Corporation  
New London, CT

The mathematical model and how to use it to analyze operational problems is described in Volume I. Volume II shows the user how to run the program and exercise all of the options. All of the information necessary to maintain the program is contained in Volume III. This includes the definition of all variables, a complete program listing, a description of every subroutine and its arguments, calls, error checking, a directory listing and the command files. The same example run is presented in all three volumes.

Requests to use PATROL should be made to:

COMMANDING OFFICER  
USCG Research and Development Center  
Avery Point  
Groton, CT 06340-6096  
(203) 441-2653

## BACKGROUND

The model described in this report was developed for the U.S. Coast Guard's Marine Vehicle Technology (MVT) program which is directed by the Marine Vehicle Technology Branch (G-DMT-2) of the Office of Research and Development in Washington, D.C. This model evolved from the original effort (Reference 1) at the Coast Guard R&D Center to develop mission measures of effectiveness (MOE's) for Coast Guard cutters. The MOE's were developed to rank order candidate vessels to replace the Coast Guard patrol boats (WPB's). To compute the MOE's a comprehensive modeling effort to accurately reflect all of the aspects of the operation was required. This will be seen in the description of the model where the vessel, area, traffic and decisions all have an effect upon the various MOE's that describe the operation. The model that has evolved has been used by the Office of Research and Development in Washington, D.C.

## INTRODUCTION

Consider the activities of a single vessel on a patrol. It leaves its home port and transits to the operating area at an "economical" speed. Upon arrival in the operating area some type of a search is initiated. This could be as simple as drifting and waiting for the traffic to pass within the radar sweep width or more complex like a bowtie search pattern. When a target is detected and classified a decision to pursue the detected vessel is made. When the subject vessel is visually identified actions may be taken to board. The results of the boarding then determine whether the vessel will be released or held for further processing. In general, some type of an escort and handing-off procedure would be required. After the hand-off, the patrolling vessel returns to the operating area and initiates the searching activity again. The patrolling vessel refuels at sea or at some local (to the operating area) base when necessary. Some combination of all of these activities takes place for the duration of time spent in the operating area. Finally, the vessel transits back to its home port and the patrol is completed.



Before we can begin to model a vessel on a patrol, some limitations and assumptions must be made. We will model the steady state performance of the patrolling vessel, i.e., the expected performance over a number of identical patrols. Although a vessel may be operating with other vessels in a fleet, there will be no interaction with these vessels (or airplanes) in this model. Some interplay may be accounted for by adjusting the model parameters. The vessel operates in a non-lethal environment, that is, it doesn't take hits that will disable or sink it. The patrolling vessel works in an area with a distribution of traffic. Some of the traffic is of interest while the rest is not. In practice, to see if a vessel of interest is smuggling or violating the law, it is necessary to board it. This will also be the case with this model. Examples of these types of operations are a blockade or barrier patrol, a fisheries patrol or anti-smuggling operations such as drug and alien interdiction.

#### MATHEMATICAL MODEL

The activities of the patrolling vessel are represented by the diagram (Figure 1). Only one of the activities can happen at a time. We will call these activities Phases of the patrol. Each phase represents a passage of time where the state of the system is essentially unchanged. The state of the system only changes when an Event occurs, i.e., phases are separated by events. For example, a Search continues until a Detection occurs. Even though the patrolling and target vessels approach each other more closely as time passes, there is no new information to act on until the detection occurs. The phases and events of the model are defined in Tables 1 and 2. When an event occurs a decision must be made as shown in Table 3. For example, when a target is approached at the end of a Pursuit, the decision must be made to board the target or not. A typical patrol scenario is presented in Figure 2.

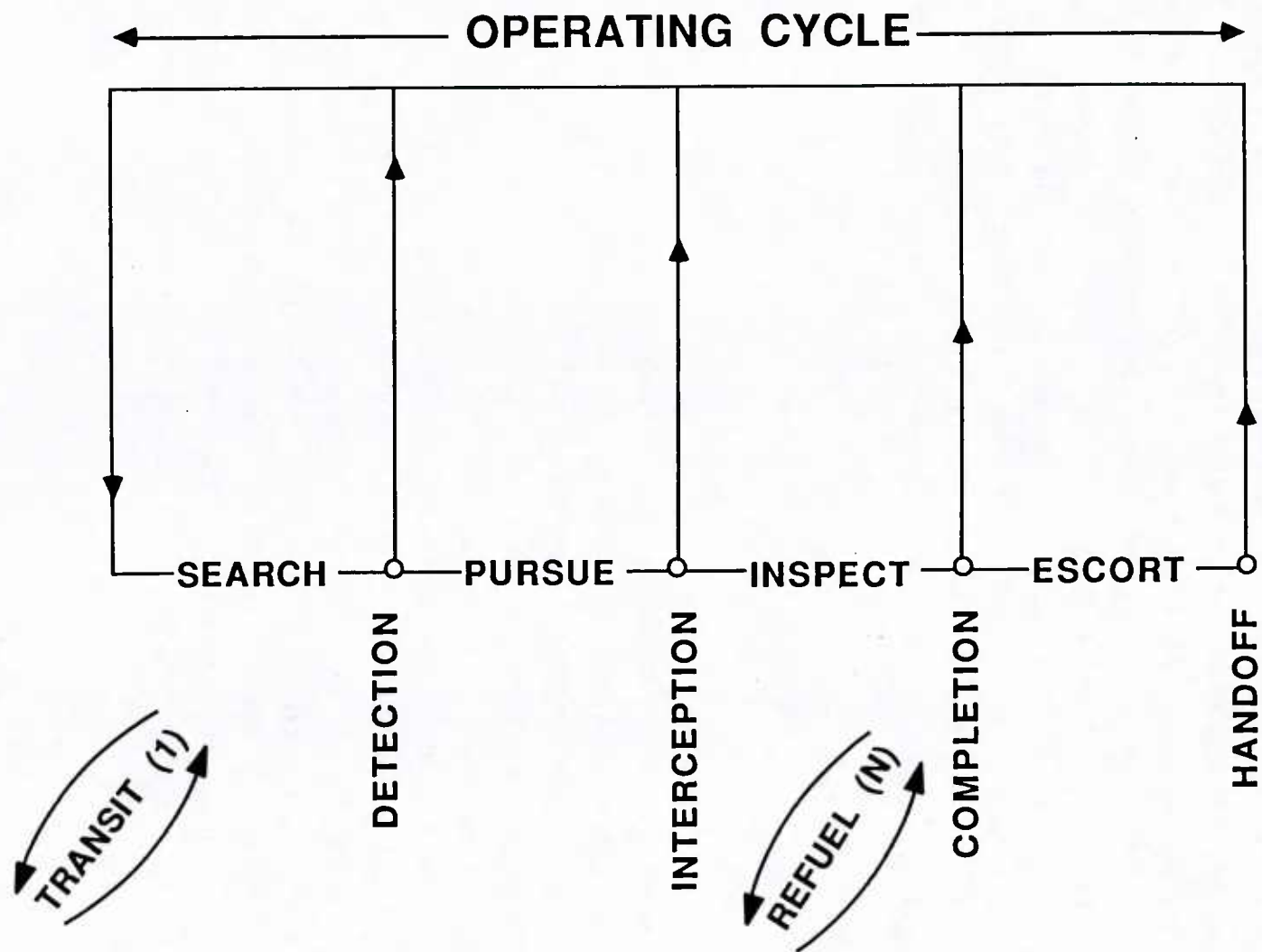


FIGURE 1. PATROL MODEL

TABLE 1 - PHASES

PHASE	ACTIVITY
SEARCH	Search for a vessel (generally using radar). Speed and search pattern chosen by the operator.
PURSUE	Chase a detected target until the subject vessel is closed to within visual range or a boarding position is achieved.
INSPECTION	Come on board and inspect the vessel. Interview the crew and/or otherwise resolve the status of the vessel.
ESCORT	Take the seized vessel to some location for further processing.
TRANSIT	Travelling to and from home port to the operating area.
REFUEL	Travelling to and from the operating area to a replenishment point and taking on fuel and stores.

TABLE 2 - EVENTS

EVENT	DEFINITION
DETECTION	A target is detected on the radar screen.
INTERCEPTION	The patrolling vessel is close enough to the target to determine if boarding is necessary.
COMPLETION	The inspection of the vessel is completed.
HANDOFF	The patrolling vessel transfers custody of the seized vessel to some other authority or otherwise disposes of the seized vessel.

TABLE 3 - DECISIONS

UPON	DECIDE TO
DETECTION	Chase the target or continue to search.
INTERCEPTION	Board the subject vessel or search for another target.
COMPLETION	Seize the vessel that was just inspected or search for another target.
HANDOFF	No decision; always return to the operating area and search.

Except for transiting and refueling, the patrolling vessel cycles through the network in Figure 1 for the duration of the patrol. We will call this network the operating cycle. In general, the average time spent in each of the phases of the operating cycle is not the same. The average time between detections could be several hours while the average intercept time might be far less (or more).

The operating cycle can be modeled as a Markov process. The phases of the patrol are the states, and the average times in each of the phases correspond to the time steps of a Markov process. Technically, this is a semi-Markov process since the mean time in each phase is different. A Markov model is completely specified by the probability transition matrix and the average time in each of the phases. As we shall see, these may be computed from the parameters associated with the particular patrol.

Consider now a way in which almost all operational problems can be organized. In Figure 3 there are four broad categories that will include all aspects of the problem. The patrolling Vessel is described by its engineering and operational characteristics. These include range, fuel consumption, speed in a seaway, sensors and other outfit, fuel tankage, physical dimensions, etc. The Area category describes the region in which the patrol takes place. Distances to home port and refuel location, size of patrol area, and sea state distribution are among the parameters that are included here. Traffic includes

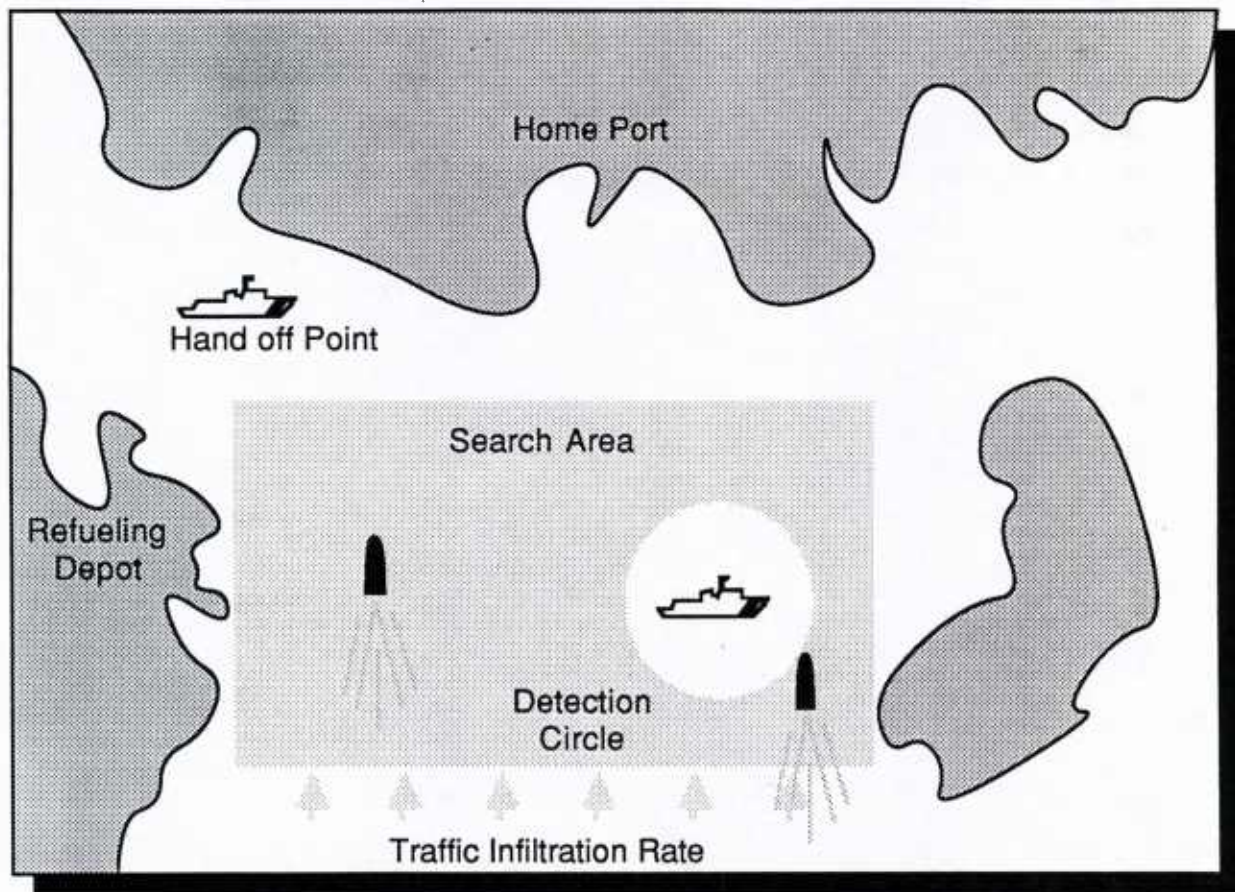


Figure 2. Typical Patrol Scenario



information on traffic density, arrival rates, course and speed, nationality, which vessels are smuggling, and so forth. Lastly, Choices represent decisions and choices at various levels of command. For instance, boarding policies, the level at which refueling is mandatory, search patterns, and the duration of the patrol are all choices made for tactical, strategic or policy reasons.

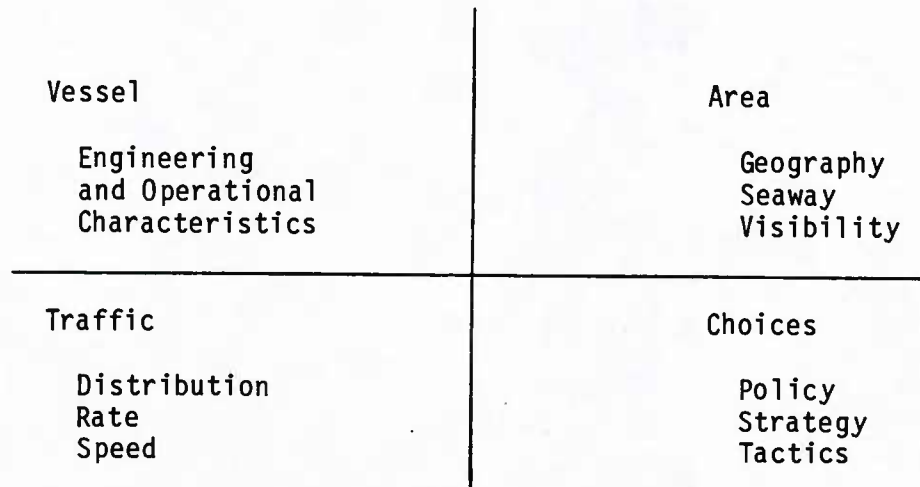


FIGURE 3. Patrol Organization

The combination of Area, Traffic, and Choices is commonly called the Scenario. Different values of parameters then correspond to different Scenarios. The usual way to analyze the performance of a vessel is to play it against a standard (or set of) Scenarios. It should be recognized that the Choices made will strongly influence a Vessel's performance. Hence, the Scenario is a prime driver in the evaluation process.

Figure 3 is an organizational schema that greatly simplifies sorting out the information that describes the patrol. The information, that describes the patrolling vessel and the scenario, is then used to produce the probability transition matrix and average times in each phase of the operating cycle for the Markov model. For example, the average time between detections can be determined if all of the required parameters are given. These include a search pattern, the size of the search area, the patrolling vessel's search speed and sweep width, the speed and arrival rates of each type of traffic in

the area, the sea state distribution, and the speed in a seaway of both the traffic and patrolling vessels. Note that it is necessary to pull information from each of the four organizational categories to compute the average time between detections. The detailed procedures used to calculate all of the average times and the probability transition matrix of the Markov model are presented in Appendix A. The development of the probability of detection and interception algorithms used in the model is presented in Appendix B.

Solving the Markov model yields the proportion of time the patrolling vessel spends in each phase of the operating cycle. The phases in the operating cycle are then merged with the transiting and refueling phases to obtain the times spent in all of the phases of the patrol. Once the total time spent in a phase of the patrol is known, the number of events is computed by dividing the time in a phase by the average time between events (the  $\mu_{ij}$  in Appendix A). The number of events, speed, distance, and vessel fuel consumption information are then used to determine when refuelings must take place. Information associated with each activity of the patrolling vessel is computed in the model. For example, the distance, time, speed (in the seaway) and fuel consumed transiting from home port to the operating area are calculated. The same information is computed for every other activity of the patrolling vessel. Details of the calculations can be found in Volume III. The information is then available to be sorted or processed further. A partial list includes:

- Searching (between detections)
- Pursuing a detected vessel
- Escorting a seized vessel to the hand-off point
- Transiting back to the operating area from the hand-off point
- Transiting to and from the refueling location
- Standing by while refueling
- Standing by while handing-off a seized vessel

Various measures of effectiveness are used to describe the performance of a vessel on a patrol. Some of the inputs to the Markov Model are indicative of

the potential performance of a vessel. The detection rate, or average time between detections, tells a lot about the follow-on events. If the detection rate is low, the number of seizures cannot be high. The total time in the operating area is sometimes used to measure deterrence. The number of vessels seized, total contraband weight shipped, seized, and missed, vessel seizure rate, and interdiction rate are all MOE's that are computed in the model.

The analyst must choose MOE's that are appropriate to the problem at hand. Detailed information available in the model may provide other information that can be further manipulated to address a particular problem.

### COMPUTER MODEL

This mathematical model, with the Markov process as the kernel, has been coded in FORTRAN and runs on a Digital Equipment Corporation VAX computer that uses the VMS\* operating system. A typical run of the program, called PATROL, is presented in Appendix C. Inputs to PATROL are organized as shown in Figure 2, but in much more detail. Outputs are organized into three major categories: Effort, Logistics and Performance. Effort shows the time spent, distance travelled, and probability of being in each of the phases of the patrol. Logistics presents mission profiles and a detailed breakout of the time, distance and fuel spent in supporting tasks such as transiting and refueling. Performance includes an event trace, average time in each phase of the operating cycle, vessel searching capabilities and mission measures of effectiveness. Also included is a traffic summary that describes the smugglers and the amount of contraband that was carried.

In addition to the standard outputs, detailed information can be printed out if desired. This includes such items as speed in the seaway of the vessel traffic in the scenario, time between refuelings, the fraction of targets able to be intercepted within a specified time and much more as seen in Appendix C.

\* Virtual Memory System. Digital Equipment Corporation references in Volume II, PATROL: A User's Guide

## USING PATROL

The utility of PATROL depends upon the application. Like any model, PATROL has its capabilities and limitations. It allows the user to perform what-if analyses by making multiple runs and varying the various parameters of the model. A utility program called VADASS\*\* (Vessel Acquisition Data Analysis Support System) is available which can help the user in this systematic variation. VADASS is a spreadsheet-based program that allows PATROL to be run repeatedly with different parameter values. Plots of the results of a number of runs can be produced.

One strength of PATROL is that information is available at various levels of detail. If the user chooses to print out all of the additional information, each item is defined in English, the computer variable name is listed and its value in physical units is given.

The outputs of PATROL are analogous to the information flow used in a systems approach to a problem. The information in Effort is equivalent to the input to the patrol. The Performance information is analogous to the output of the patrol while Logistics information reflects the cost of doing business, i.e., supporting that level of effort and performance. This can be seen in Figure 4.

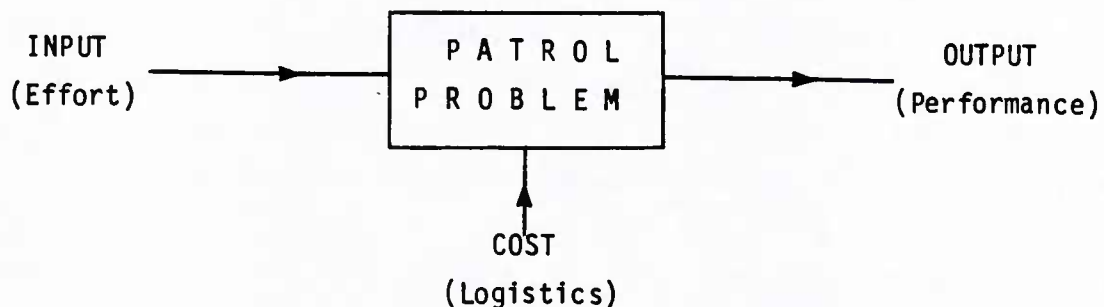


FIGURE 4. Systems Approach to the Problem

\*\* VADASS is maintained at the Coast Guard R&D Center in Groton. Refer to page 1 for requesting information.

Further insight into particular patrolling problems may be gained by manipulating the various factors available using input-output or cost-performance/cost-benefit approaches.

### EXAMPLE

Refer to the model run in Appendix C for this example. We assume that we have an operational problem that is appropriately modeled using the assumptions on page 3. We are using a 155 foot, 32 knot semi-planing boat, much like the French La Combattante, for a law enforcement patrol. The vessel is described by its engineering and operational characteristics. These are the Inputs for Cutter screens 1, 2 and 3.

We will use the following scenario. The cutter is homeported 125 miles away from the operating area. Refueling is done from a High Endurance Cutter (WHEC) which will always (for our purposes) be on-scene, but on the average 30 miles away from the patrolling cutter. A seized vessel will be handed off to a Patrol Boat (WPB) that will come out to meet the cutter. The average distance to escort a seized vessel to meet the WPB is 50 miles.

We input this distance information in screen 1 of the scenario. The vessel traffic in the region is described in screens 2 and 3. The choices that we make for this patrol are shown in screens 4 through 6. For example, on screen 4, the entire patrol will last ten days and we choose not to chase any detected vessel that requires more than 1.5 hours to catch. The information on screen 5 is the percent of each class/type vessel that we choose to pursue after detecting, and board after catching. Although we don't have any real control over some of the parameters such as the time for a statement of No Objection (SNO), the times on screen 6 are all grouped together so we may understand how they influence the operational performance. We do have control over when we refuel and how much fuel is on board when we leave homeport.



The three PATROL RESULTS pages present a high level description of this operation. The PRE MARKOV, MARKOV, and POST-MARKOV pages present all of the detailed information that is computed in the model. In many applications VESSEL PERFORMANCE (in the mission) will be the most interesting page of PATROL RESULTS. The event trace shows the sequence of events of interest on the patrol. AVERAGE TIMES are inputs to the model but are also indicators of potential performance. The CUTTER SEARCHING CAPABILITY (the POD's) are indications of the fraction of targets that the cutter can detect in each phase of the patrol. However, the cutter only pursues traffic detected while in the search phase. The MOE's are ratios of favorable and total occurrences (e.g. the number of seizures divided by the total number of vessels smuggling). We can analyze these numbers further to glean more information about the patrol. If we were interested in the average tonnage of contraband carried by a smuggler, we simply divide the total tonnage shipped on the 10 day patrol by the total number of smugglers, to obtain 2.14 tons/smuggler.

The user must make multiple runs of the model to see the effect of any parameter or set of parameters. For example, to better understand the effect of stockpiling seized vessels or forward positioning the cutter, runs would be made with appropriate distances for escorting and transiting. It may also be necessary to modify the processing time of the seized vessel when they are stockpiled. If you choose to stockpile, this would indicate that there is another cutter in the area. You might possibly want to change your refueling level to some lower value since transits will be much shorter distances. To use PATROL to address any issue requires the user to think about the problem in light of all of the available parameters of PATROL.

## REFERENCES

1. Development of Measures of Effectiveness for Marine Vehicles for Coast Guard Missions. Tedeschi, L., United States Coast Guard Office of Research and Development Report CG-D-41-82,. July 1982. Available through the National Technical Information Service, Springfield, VA, ADA 133 622.
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4. Naval Operations Analysis. Naval Institute Press, 1968.

## APPENDIX A

### DEVELOPMENT OF THE MARKOV MODEL

## APPENDIX A

The following development will make extended use of indices so it is best that we face up to the notation now. The row index,  $i$ , of a matrix will identify a nationality and whether it is of special interest (suspicious) or not. The column index,  $j$ , identifies a type of vessel. We will call items referred to by  $i$  and  $j$  as class and type, respectively. The following table shows this more clearly.

<u>Index</u>	<u>Description</u>
$i = 1$	Class = U.S. Special Interest
$i = 2$	Class = U.S. Other
$i = 3$	Class = Foreign Special Interest
$i = 4$	Class = Foreign Other
$j = 1$	Type = Merchant Ship
$j = 2$	Type = Fishing Boat
$j = 3$	Type = Pleasure Craft
$j = 4$	Type = Fast Target

The notation  $j$  will be used for partial summing.

where  $j \in \equiv j \in U_j < V$ .

This reads as follows: Sum over those vessel types (values of  $j$ ) that are slower (speed  $U_j$ ) than the pursuing vessel (speed  $V$ ). In other words, only consider the traffic that you can catch.

One of the fundamental inputs to the Markov model is the traffic description, primarily the speeds and arrival rates of each class and type vessel. Another important set of parameters are those involving decisions, e.g., which vessels are pursued and for how long, which ones are boarded, etc. All of the following computations that involve rates (with the exception of tonnage) are expressed in units of events per hour.

The arrival rate (in the operating area) of a vessel of class  $i$  and type  $j$  is  $r_{ij}$ . The Vessel Arrival Rate of the  $j$  type vessel,  $r_j$ , is the sum over all classes  $i$ .

$$r_j = \sum_{i=1}^4 r_{ij}$$

The overall arrival rate,  $\dot{R}$ , of all of the traffic in the area is the sum of all of the individual arrival rates.

$$\dot{R} = \sum_{j=1}^4 r_j$$

The overall detection rate,  $\dot{D}$ , requires the probability of detection,  $POD_j$ , of each type of vessel.

$$\dot{D} = \sum_{j=1}^4 r_j \cdot POD_j$$

Not all vessels that are detected will be pursued. Some targets can be classified as of no interest.  $I_j$  represents the fraction that will be pursued. Other targets cannot be caught within some time  $\tau$ , the fraction that can be is  $k_j$ . And, some targets are faster than the pursuing vessel, so here we use the  $j \in$  notation. Now the interception rate,  $\dot{I}$ , is

$$\dot{I} = \sum_{j \in} r_j \cdot POD_j \cdot I_j \cdot K_j$$



Boarding a vessel is a matter of choice. The variable  $b_{ij}$  is the fraction of class  $i$ , type  $j$  vessels that is boarded that reflects that choice. Boarding is curtailed when the wave height exceeds a certain value which we will call  $\text{Max } h$ . The integral of the sea state distribution curve up to  $\text{Max } h$  is the fraction of time that boarding may take place. The boarding rate,  $\dot{B}$ , is

$$\dot{B} = \sum_{j \in} \sum_{i=1}^4 b_{ij} \cdot r_{ij} \cdot \text{POD}_j \cdot I_j \cdot K_j \cdot C$$

$$\text{where } C = \int_0^{\text{Max } h} p(h) dh \quad 0.0 \leq C \leq 1.0$$

Only violators are seized.  $s_{ij}$  represents the fraction of each class/type that are violators. The seizure rate of vessels,  $\dot{S}_v$  is

$$\dot{S}_v = \sum_{j \in} \sum_{i=1}^4 s_{ij} \cdot b_{ij} \cdot r_{ij} \cdot \text{POD}_j \cdot I_j \cdot K_j \cdot C$$

Many other quantities of interest can also be calculated.  $t_{ij}$  represents the amount of contraband (in tons) carried by each class/type combination. The tonnage seizure rate  $\dot{T}_{sz}$  is

$$\dot{T}_{sz} = \sum_{j \in} \sum_{i=1}^4 t_{ij} \cdot s_{ij} \cdot b_{ij} \cdot r_{ij} \cdot \text{POD}_j \cdot I_j \cdot K_j \cdot C$$

Now we will address some other arrival rates. The smuggler arrival rate,  $\dot{SR}$ , is

$$\dot{SR} = \sum_{j=1}^4 \sum_{i=1}^4 s_{ij} \cdot r_{ij}$$

The tonnage arrival rate in the area,  $\dot{TR}$ , is

$$\dot{TR} = \sum_{j=1}^4 \sum_{i=1}^4 t_{ij} \cdot s_{ij} \cdot r_{ij}$$

The fractions boarded and seized of both U.S. and foreign vessels are BUS, BFor, SUS, and SFor, respectively.

$$BUS = \frac{\sum_{i=1}^2 \dot{B}_i}{\dot{B}}$$

$$BFor = 1 - BUS \quad \text{or} \quad \frac{\sum_{i=1}^4 \dot{B}_i}{\dot{B}}$$

$$SUS = \frac{\sum_{i=1}^2 \dot{s}_{vi}}{\dot{s}_v}$$

$$S_{For} = 1 - S_{US} \text{ or } \frac{\sum_{i=3}^4 \dot{S}_{vi}}{\dot{S}_{vi}}$$

Here the summation is over the components of the function in the denominator.

Not all of these quantities are used in computing the inputs to the Markov model, but it is convenient to discuss them as a group. Some are used inside the model and others are used in MOE calculations.

Now we are ready to compute the probability transition matrix. We first check to make sure that

$$\dot{R} \geq \dot{D} \geq \dot{I} \geq \dot{B} \geq \dot{S}_v$$

This must be true for the physical problem. You can't seize more vessels than you board. You can't board more vessels than you intercept, etc.

The transition matrix that represents the network in Figure 1 is

$$P = \begin{bmatrix} P_{11} & P_{12} & 0 & 0 \\ P_{21} & 0 & P_{23} & 0 \\ P_{31} & 0 & 0 & P_{34} \\ P_{41} & 0 & 0 & 0 \end{bmatrix}$$

$$P_{12} = \frac{\dot{I}}{\dot{D}} \quad P_{11} = 1 - P_{12}$$

$$P_{23} = \frac{\dot{B}}{\dot{I}} \quad P_{21} = 1 - P_{23}$$

$$P_{34} = \frac{\dot{S}_V}{\dot{B}} \quad P_{31} = 1 - P_{34}$$

$$P_{41} = 1.0 \text{ and } P_{ij} = 0.0 \text{ Otherwise}$$

Each  $P_{ij}$  is the ratio of rates. If we multiply the top and bottom by an increment of time  $T$  this gives the number of events of each type. It is this ratio of events which is represented by the probability. The  $T$ 's cancel leaving the simple computational form above.

The average times in each phase are designated as  $\mu_j$  by convention. (Reference 3)

Average Time	Notation
Between Detections	$\mu_1$
To Intercept	$\mu_2$
To Inspect	$\mu_3$
To Escort (& Return)	$\mu_4$

$$\mu_1 = \frac{1}{\dot{D}}$$

$$\mu_2 = \sum_{j \in} r_j \cdot \text{POD}_j \cdot I_j \cdot K_j \cdot \bar{t}_j^{\text{Int}}$$

$$\mu_3 = \text{BUS} \times t_{\text{US}}^{\text{B/I}} + \text{BFor} \times [\text{SNOI} + t_{\text{For}}^{\text{B/I}}]$$

$$\mu_4 = \frac{\sum_{j \in} \sum_{i=1}^4 s_{ij} \cdot b_{ij} \cdot r_{ij} \cdot \text{POD}_j \cdot I_j \cdot K_j \cdot C \cdot \bar{t}_j^{\text{Esc}}}{\dot{S}_V}$$

where

$$\begin{aligned} \bar{t}_j^{\text{Int}} &= \text{Average time to intercept the } j\text{th type of vessel.} \\ \bar{t}_j^{\text{B/I}} &= \text{Average time to board and inspect the } j\text{th type of U.S. or Foreign vessel.} \\ \bar{t}_j^{\text{Esc}} &= \text{Average time to escort the } j\text{th type vessel to a handoff location and transit back to the operating area.} \\ \text{SNO} &= \text{Time for a Statement of No Objection to board and inspect.} \\ \bar{t}_j^{\text{Esc}} &= D_E \left( \frac{U_j + V}{U_j \cdot V} \right) + t_{\text{Handoff}} \\ D_E &= \text{Distance to escort.} \end{aligned}$$

The solution of the Markov model involves two steps. First, the probabilities ( $\pi_i$ 's) for the embedded chain are found. Next, these results (the  $\pi_i$ 's) are weighted by the  $\mu_i$ 's to obtain the probabilities ( $P_i$ 's) of being in each of the four phases of the patrol.

The Embedded Chain Solution is

$$\pi_1 = \frac{1.0}{(1 + P_{12} (1 + P_{23} (1 + P_{34})))}$$

$$\pi_2 = P_{12} \cdot \pi_1$$

$$\pi_3 = P_{23} \cdot \pi_2$$

$$\pi_4 = P_{34} \cdot \pi_3$$

The probability  $P_i$  of being in the  $i$ th phase of the patrol is

$$P_i = \frac{\mu_i \cdot \pi_i}{\sum_{i=1}^4 \mu_i \cdot \pi_i} \quad i = 1, 2, 3, 4$$

The  $P_i$ 's are then multiplied by the total time on-scene,  $T_{os}$ , to obtain the total time  $T_i$  in each of the four phases of the operating cycle, searching, pursuing, inspecting and escorting. The  $T_i$  for each phase is divided by the average time per event to obtain the total number of events in that phase of the patrol. For example,

$$\# \text{ Detections} = \frac{T_1}{\mu_1} = \frac{T_{os} \cdot P_1}{\mu_1}$$



## APPENDIX B

### DETAILS OF PROBABILITY OF DETECTION AND INTERCEPTION CALCULATIONS

## APPENDIX B

### DETAILS OF PROBABILITY OF DETECTION AND INTERCEPTION CALCULATIONS

#### DETECTION

A single vessel is searching for targets in a region with a uniform distribution of traffic. Each vessel type has a (potentially) different speed. If the operator of the searching vessel chooses to search in a bowtie pattern, it is not clear what the angle for the bowtie should be. The textbook (Reference 4) formulation is that the angle is  $\arcsin(u/v)$ . This then leads to the equation for the probability of detection (POD) shown below.

$$POD_{\infty} = \frac{1}{\lambda + 1} \left[ 1 + \frac{\gamma \sqrt{\gamma^2 - 1}}{\gamma + 1} \right]$$

where      $\gamma$      =  $V/U$ , speed ratio  
                $U$      = Target speed  
                $V$      = Searcher speed  
                $\lambda$      =  $D' / W$   
                $W$      = Sweep Width  
                $D$      = Channel Width (Barrier Size)  
                $D'$     =  $D - W$

The function of  $\lambda$  on the outside of the brackets is the zero speed POD. The function of  $\gamma$  inside the brackets is the effect of speed upon the POD. Some values are shown below:

<u>Speed Ratio</u>	<u>Speed Effect</u>
1.00	0.0
1.25	0.4166
1.50	0.6708
1.75	0.9139
1.8393	1.0000
2.00	1.1547
2.25	1.3954
2.50	1.6366

<u>Speed Ratio</u>	<u>Speed Effect</u>
2.75	1.8786
2.8752	2.0000
3.00	2.1213

We see that when the searcher is 1.8393 times faster than the target, that the POD is twice as great as if the searcher simply drifted in the barrier and waited for the target to come to him.

This formulation for POD is based upon a cookie cutter model of search. In this class of models, everything within the sweepwidth is detected (POD = 1.0) and everything outside of the sweepwidth is not detected (POD = 0.0). The sweepwidth is usually taken as the integral of the lateral range curve. The probability of detection for the barrier is determined by taking the ratio of the area swept by the searcher to the total area in which the target may be found. The development of the formula in Reference 4 used rectangular sections for each search leg. This did not account for the circular ends of the cookie cutter. If we do, the POD is reduced slightly to become

$$POD_{\Delta} = \frac{1}{\lambda + 1} \left[ 1 + \frac{\gamma \sqrt{\gamma^2 - 1}}{\gamma + 1} \left( 1 - \frac{0.1073}{\lambda} \right) \right]$$

The above formulation is only valid when the relationship of the searching vessel speed and the target vessel speed is  $\alpha = \sin^{-1} (U/V)$ . If another target vessel with a different speed is involved, another POD function must be formulated.

There are three cases that can occur in practice

$$U_1 > V \sin \alpha$$

$$U_1 = V \sin \alpha \quad \alpha = \text{angle with barrier}$$

$$U_1 < V \sin \alpha$$

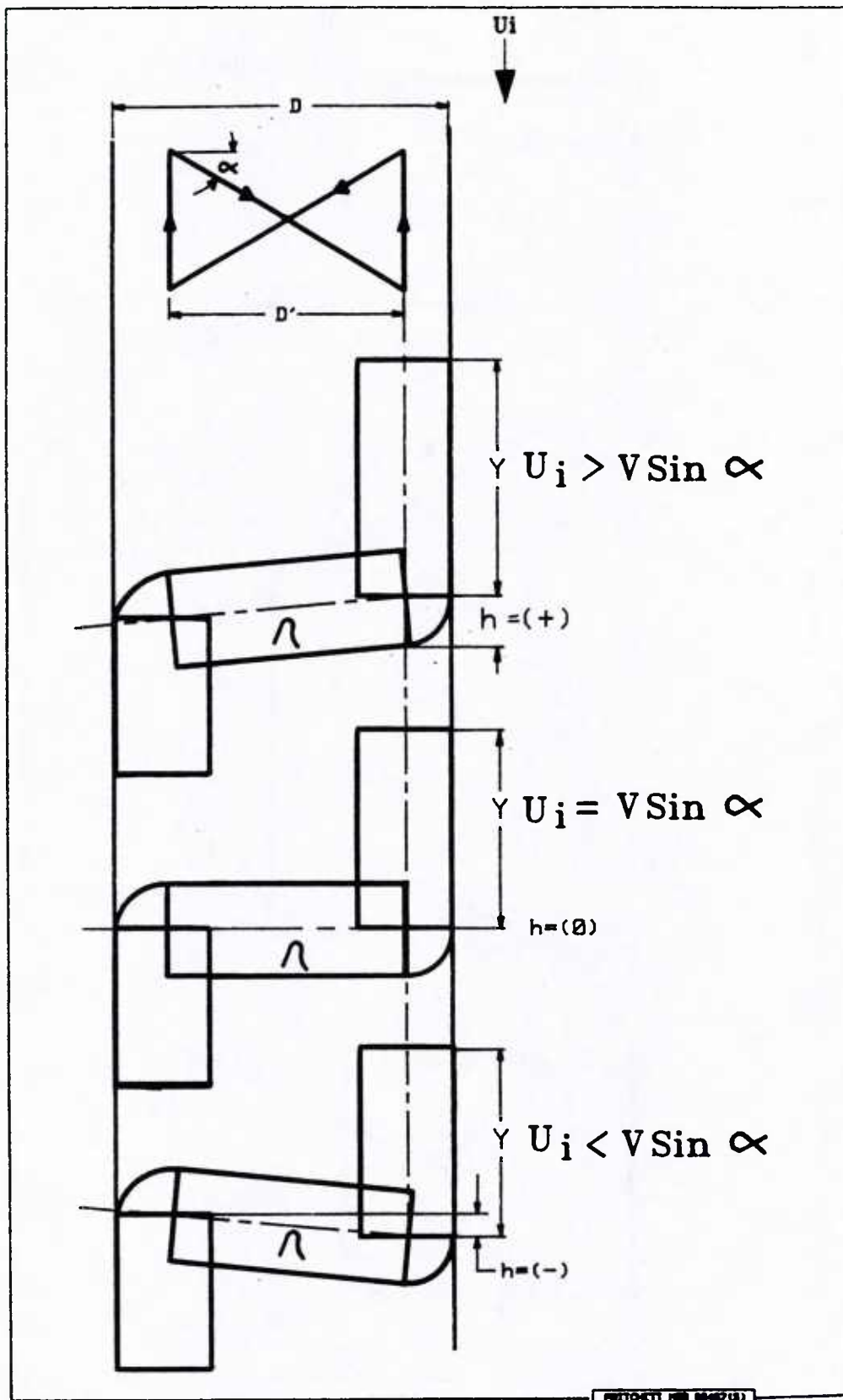


FIGURE B1. THREE CASES OF BOWTIE SEARCH PATTERN.

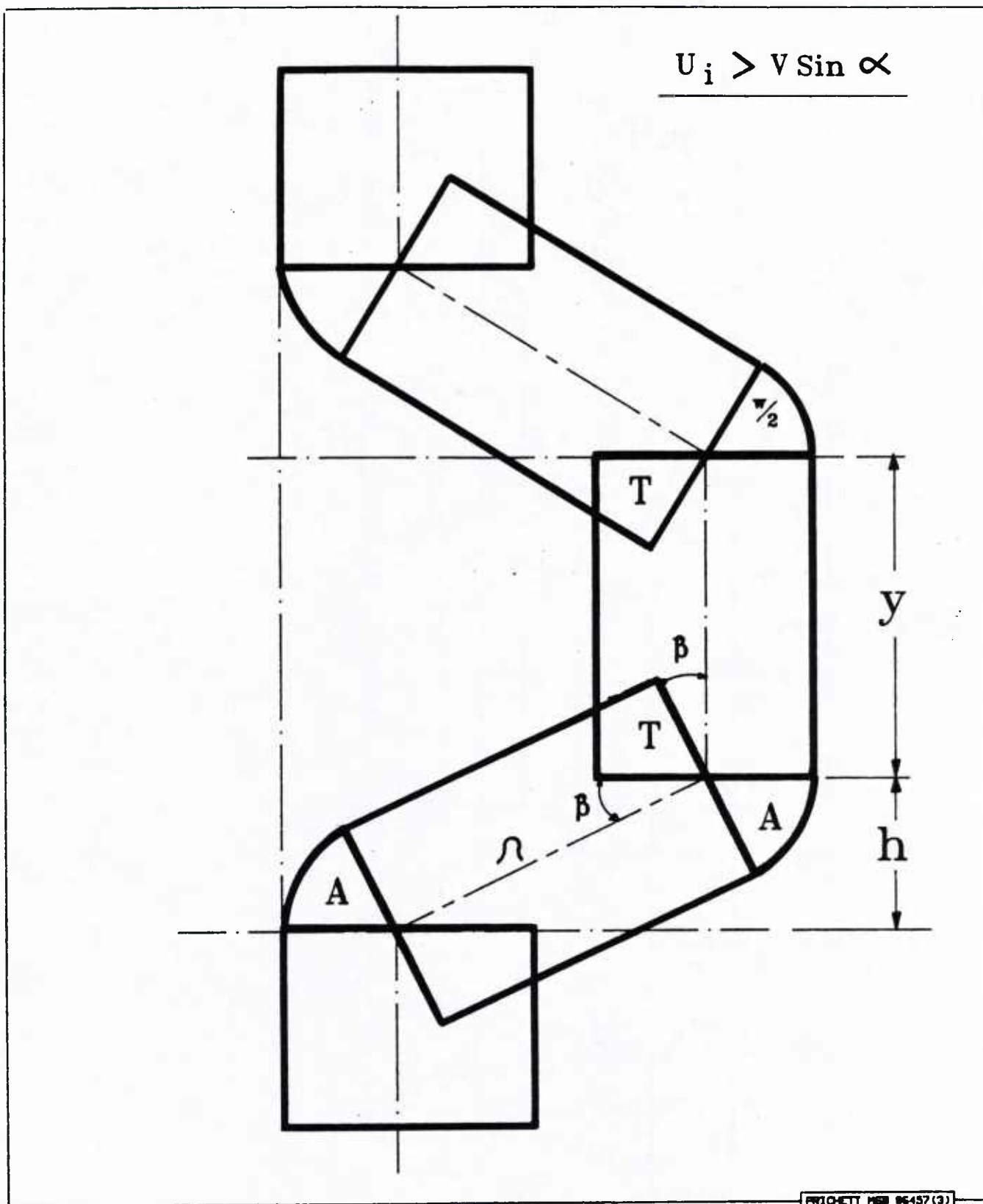


FIGURE B2. DETAILS OF SEARCH PATTERN WHEN  $U_i > V \sin \alpha$ .

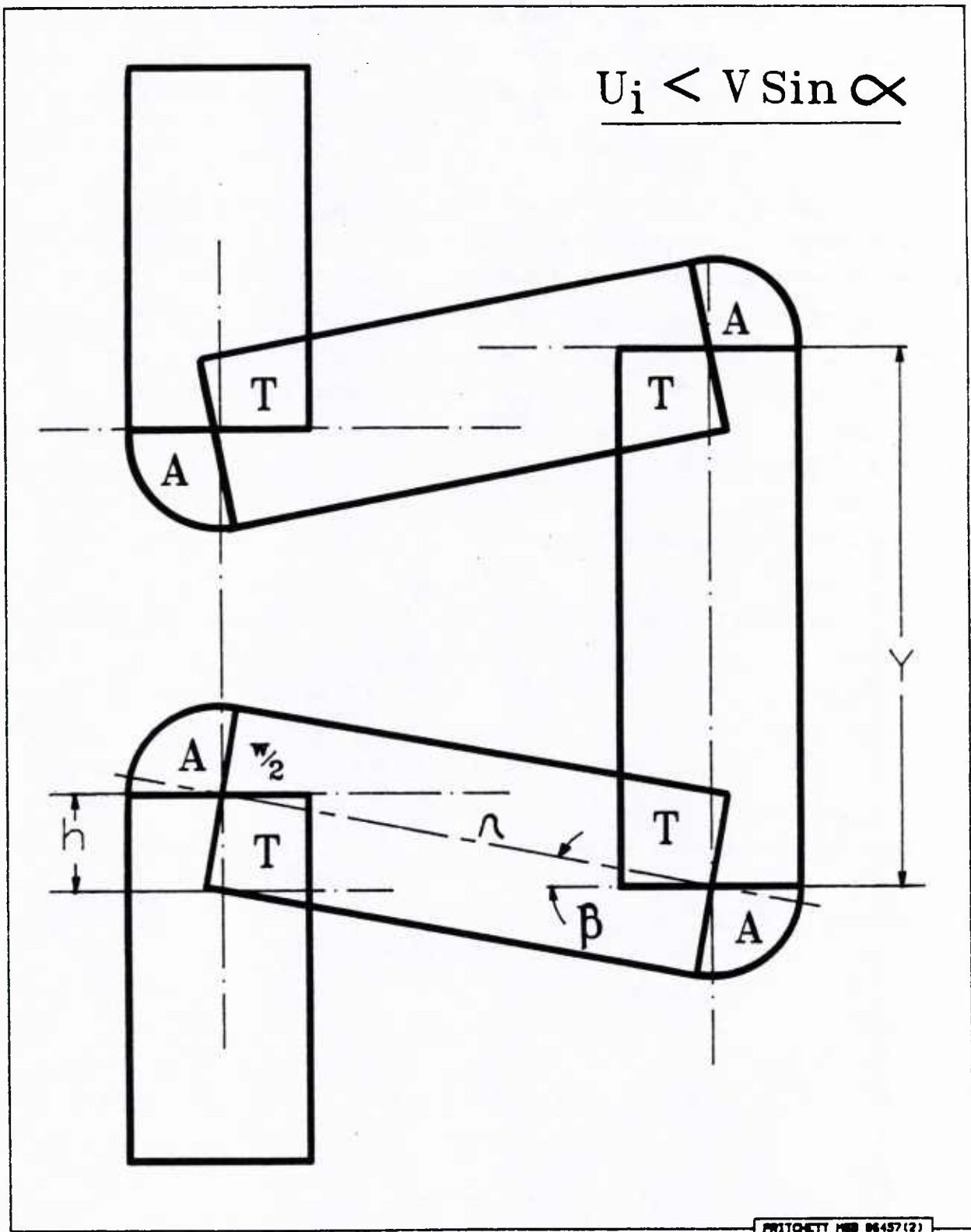


FIGURE B3. DETAILS OF SEARCH PATTERN WHEN  $U_1 < V \sin \alpha$ .



In PATROL, the angle  $\alpha$  is determined by using the average speed of the traffic  $\bar{U}$ . Then  $\alpha = \sin^{-1} (\bar{U}/V)$ . When  $U_i = \bar{U}$ , we can use the standard formulation, otherwise we compute the POD using the same cookie cutter approach.

When  $U_i > V \sin \alpha$  we expect a lower POD and when  $U_i < V \sin \alpha$  we expect a higher POD than the standard form. In Figure B-1, we see the bowtie search pattern of the Coast Guard cutter in earth centered coordinates. Then each of the three cases is shown in body centered coordinates to be consistent with Reference 4. To compute the POD, we will use one half of a cycle of the bowtie pattern since the other half is the mirror image. The target is equally likely to be in any part of the rectangle of width D and height  $y \pm h$  (depending upon the case). In the intercept computations that follow we will need the individual POD contributions from the diagonal and parallel legs so we will keep their identities.

The only complicated thing involved in the POD calculations is keeping track of the small geometric pieces that make up the swept areas. These are shown in Figures B-2 and B-3. The formulation for each term is presented in Table 4. In this coordinate system,  $h$  and  $\beta$  can take on positive or negative values. In Case 1 ( $U_i > V \sin \alpha$ )  $h$  and  $\beta$  are both positive while in Case 3 ( $U_i < V \sin \alpha$ )  $h$  and  $\beta$  are both negative. The total POD is the sum of the POD contributions from the parallel and diagonal search legs.

#### POD FORMULAS AND GEOMETRY

$$y = \frac{D' \tan \alpha}{V} (U_i + V) \quad n \text{ mi.}$$

$$h = \frac{D' \sqrt{1 + \tan^2 \alpha}}{V} (U_i - V \sin \alpha) \quad n \text{ mi.}$$

$$r = D' \sqrt{1 + \left( \frac{1 + \tan^2 \alpha}{V^2} \right) (U_i - V \sin \alpha)^2} \quad n \text{ mi.}$$

$$\beta = \tan^{-1} \left[ \frac{\sqrt{1 + \tan^2 \alpha} (U_i - V \sin \alpha)}{V} \right] \quad \text{Radians}$$

$$A = \frac{w^2}{4} \cdot \frac{1}{2} \left[ \frac{\pi}{2} - \beta \right] \quad \text{n mi.}^2$$

$$T = \frac{w^2}{4} \left[ \tan \left( \frac{\pi}{4} - \frac{\beta}{2} \right) \right] \quad \text{n mi.}^2$$

$$POD_P = \frac{w \cdot y + A - T}{(D' + w) (y + h)} \quad \#$$

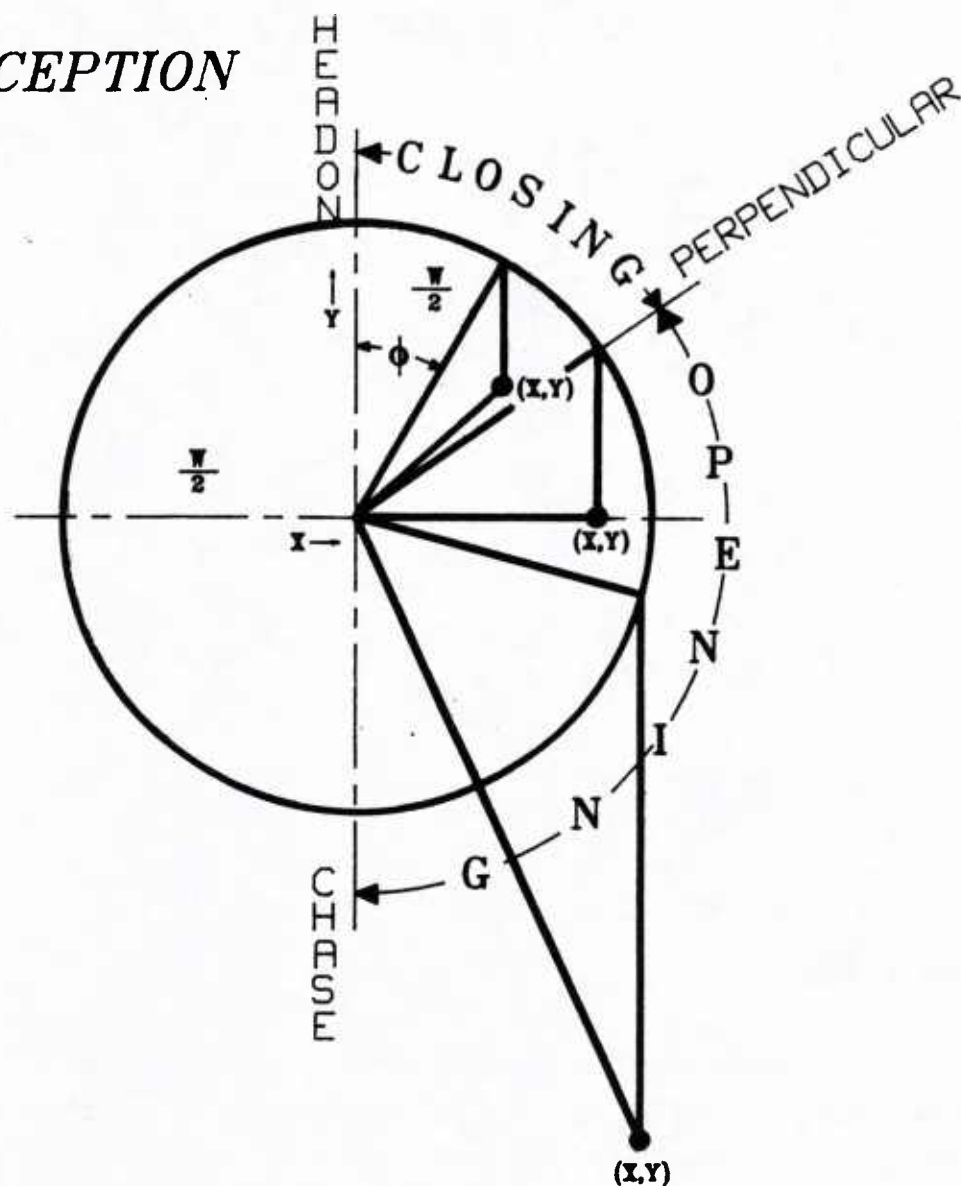
$$POD_D = \frac{w \cdot r + A - T}{(D' + w) (y + h)} \quad \#$$

$$POD_T = \frac{w(y + r) + 2(A - T)}{(D' + w) (y + h)} \quad \#$$

### INTERCEPTION

In Figure B-4, we see the geometry of all possible detections in the cookie cutter model. Detections take place at the edge of the circle. In the model, once the target is detected, the searching vessel makes an instantaneous change of course and speed (scenario screen 4) to intercept the target in the minimum time. The target continues in the same direction but changes to the evasion speed (scenario screen 2). The pursuing vessel's velocity vector has two components. One is parallel and the other is perpendicular to the target vessel's velocity vector. Detections within the CLOSING arc are intercepted with the parallel component of the intercepting vessel's velocity vector pointing in the opposite direction of the evading vessel's velocity vector. Detections within the OPENING arc have the parallel component of the velocity vector of the intercepting vessel going in the same direction as the evading vessel's velocity vector. The PERPENDICULAR point is when there is no parallel velocity component for the pursuing vessel.

# INTERCEPTION



$\phi$  = Measured clock wise from vertical to the detection vector ( $w/2$ ).

$w$  = Sweep Width

$T \equiv \frac{w/2}{U} =$  Basic time unit for Evader to travel the distance of the detection vector.

$\gamma = \frac{v}{u} =$  Speed ratio ( $\gamma > 1.0$ ).

X&Y = Coordinates of intercept point.

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FIGURE B4. INTERCEPTION GEOMETRY.

We see that the intercept times range from a minimum of  $(t/T)/(\gamma + 1)$  to  $(t/T) / (\gamma - 1)$  but the times do not vary linearly with  $\gamma$ . The intercept time is the solution of the triangle shown in Figure B-4. The ratio of maximum to minimum intercept times as a function of the speed ratio  $\gamma$  is presented in Table B-I.

TABLE B-I

Speed Ratio	Max Intercept Time
	Min Intercept Time
1.0	--
1.125	17.00
1.25	9.00
1.50	5.00
1.75	3.67
1.8393	3.38
2.0	3.00
2.25	2.60
2.50	2.33
2.75	2.14
2.8752	2.07
3.0	2.00
10.0	1.22

At the point of interception,

$$x = \frac{W}{2} \sin \phi$$

$$y = \frac{W}{2} \cos \phi - ut$$

and

$$r = \sqrt{x^2 + y^2}$$

The time to intercept,  $t$ , is

$$t = T \left[ \frac{-\cos \phi + \sqrt{\gamma^2 - (1 - \cos^2 \phi)}}{(\gamma^2 - 1)} \right]$$

Using a non-dimensional form, the intercept time for some particular values of  $\phi$  are

$\phi$	$t/T$
0	$\frac{1}{\gamma+1}$
$\frac{\pi}{4}$	$\frac{-1 + \sqrt{2\gamma^2 - 1}}{\sqrt{2}(\gamma^2 - 1)}$
$\tan^{-1}(\frac{v}{u})$	$\frac{1}{\sqrt{\gamma^2 + 1}}$
$\frac{\pi}{2}$	$\frac{1}{\sqrt{\gamma^2 - 1}}$
$\cos^{-1}(\frac{2 - \gamma^2}{2})$	1
$\frac{3\pi}{4}$	$\frac{1 + \sqrt{2\gamma^2 - 1}}{\sqrt{2}(\gamma^2 - 1)}$
$\pi$	$\frac{1}{\gamma - 1}$

To catch the target in one basic time unit requires a speed ratio at least as great as shown for various values of  $\phi$ .

$\phi$	$\gamma$
$\frac{\pi}{2}$	$\sqrt{2}$
$\frac{2\pi}{3}$	$\sqrt{3}$
$\frac{3\pi}{4}$	$\sqrt{2 + \sqrt{2}}$
$\frac{5\pi}{6}$	$\sqrt{2 + \sqrt{3}}$
$\pi$	2

The locus of points (X, Y) then define a curve on which the intercept must take place. To produce this curve,  $\phi$  is simply varied from  $0^\circ$  to  $180^\circ$  and X and Y are computed. Interesting values of  $\phi$  such as

$$\tan^{-1}(\gamma) \text{ and } \cos^{-1}\left(\frac{2-\gamma}{2}\right) \text{ can be determined}$$

and then plugged into the equations above to determine their associated intercept locations. Given W, U and  $\gamma$  the intercepting solution, (t, X, Y and r) is completely specified. Being able to compute the intercept time for any combination of sweepwidth, speeds and detection angle allows the operator or analyst to ask and answer some interesting questions.

Based upon particular aspects of the patrol, there may well be a maximum acceptable intercept time. Let's call it  $\tau$ . We can then solve for  $\tau$  in terms of the detection vector angle  $\phi$ . This limiting value  $\phi_L$  is maximum value of  $\phi$  for which the searching vessel would choose to intercept the detected vessel. The equation that describes the intercept, is

$$\frac{W}{4} + U^2 t^2 - 2 \cdot \frac{W}{2} U t \cos \phi = V^2 t^2$$

It is solved for  $\phi$

$$\phi = \cos^{-1} \left[ \frac{1}{2} \left( \frac{T}{t} - \frac{t}{T} (\gamma^2 - 1) \right) \right]$$

$$\text{where } T = W/2U$$

, Then plugging in  $\tau$  for t yields

$$\phi_L = \cos^{-1} \left[ \frac{1}{2} \left( \frac{T}{\tau} - \frac{\tau}{T} (\gamma^2 - 1) \right) \right]$$

$\phi_L$  is the maximum angle at which an interception can be made in less than  $\tau$  hours.



## AVERAGE INTERCEPT TIME

The average intercept time is the average of all intercepts that can be done in a time less than or equal to the maximum acceptable intercept time  $\tau$ . A greater fraction of the vessels detected on the parallel leg than on the diagonal leg can be intercepted within  $\tau$  hours. First we choose  $\tau$  and compute  $\phi_L$  (see page B-11).

Parallel Leg - No matter what the relationship ( $>$ ,  $=$ , or  $<$ ) between  $U_i$  and  $V \sin \alpha$  ( $\alpha$ , the angle the diagonal search leg makes with the barrier), all of the detections are made on the upper half of the circle. The searcher and the evader are approaching each other "head-on" as shown below. The targets are assumed to be uniformly distributed over the channel. They must be mapped onto the detection circle to determine the average intercept time.

If  $\phi_L > \pi/2$ , the maximum intercept time is not in the upper half of the circle. So we simply set  $\phi_L = \pi/2$ . When  $\phi_L < \pi/2$  we have the diagram shown below.

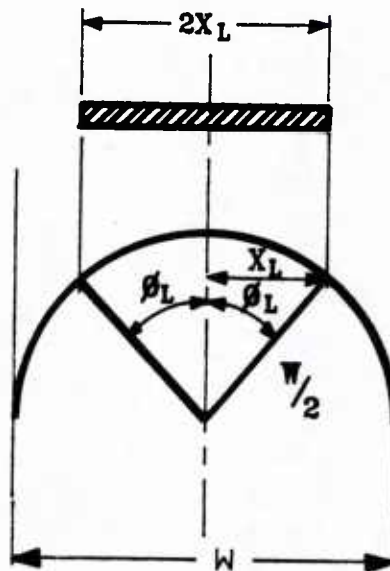


DIAGRAM 1

Since the problem is symmetrical, we will use only one half of the semicircle for the following development. Next, the value of  $X_L$  is determined:

$$X_L = \frac{W}{2} \sin \phi_L$$

The target density is broken up into N equal pieces and the center of each piece is projected onto the detection circle. The detection angle associated with the ith piece is

$$\phi_i = \sin^{-1} \left[ \left( \frac{i-1/2}{N} \right) \right] \sin \phi_L$$

The 1/2 is in the equation because we are mapping the center of the increment of target density into the detection circle.

Next,  $t_i$  is calculated for each  $\phi_i$

$$t_i = T \left[ \frac{-\cos \phi_i + \sqrt{\gamma^2 - (1 - \cos^2 \phi_i)}}{(\gamma^2 - 1)} \right] \quad i = 1, 2, \dots, N$$

The N  $t_i$ 's are then averaged to get the average intercept time on the parallel leg  $\bar{T}_I^P$ .

Diagonal Leg - In the diagonal leg of the search pattern, detections can be made on both the upper and lower part of the detection circle. Although there are three cases ( $U_i > , =, < V \sin \alpha$ ), they will reduce to one computational form. First, refer back to Figure B-1 to see the position of the diagonal search leg for each case. We will begin with Case II since that is the easiest.

Case II ( $U_i = V \sin \alpha$ ). Note the following diagram. Targets detected up to

the angle  $\phi_L$  can be intercepted within  $\tau$  hours. We proceed as before, i.e. by mapping a uniform target distribution onto the detection circle between 0 and  $\phi_L$ .

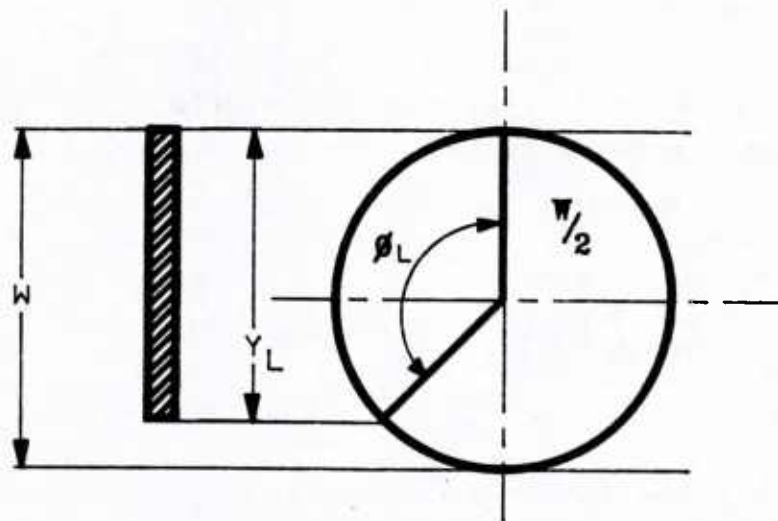


DIAGRAM 2

The value of  $Y_L$  is

$$Y_L = \frac{W}{2} (1 - \cos \phi_L)$$

The uniform distribution of targets is broken up into  $N$  parts and  $\phi_i$  is calculated for each.

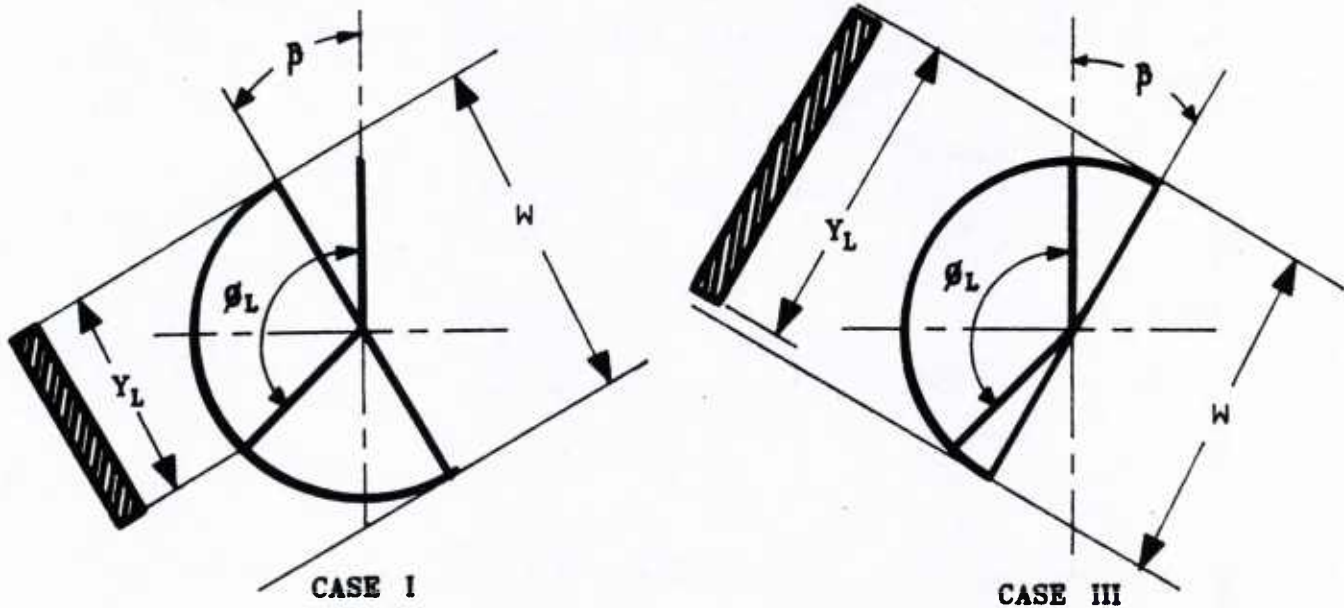
$$\phi_i = \cos^{-1} \left[ 1 - \frac{(i - 1/2)}{N} (1 - \cos \phi_L) \right] \quad i = 1, 2, \dots, N$$

Next, the intercept time for each  $t_i$  is calculated.

$$t_i = T \left[ \frac{-\cos \phi_i + \sqrt{\gamma^2 - (1 - \cos^2 \phi_i)}}{(\gamma^2 - 1)} \right] \quad i = 1, 2, \dots, N$$

The  $t_i$ 's are then summed and divided by the total number ( $N$ ) to obtain the average intercept time for the diagonal leg,  $T_I^D$ .

Cases I and III - Referring back to Figure B-1, we see that the diagonal leg of the search pattern is rotated up or down with respect to the horizontal axis through an angle  $\beta$ . This leads to the following two diagrams.



The value of  $\phi_L$  is the same in both cases but the cross section of targets is reduced or expanded due to the rotation  $\beta$ .  $Y_L$  has the same form in both cases, i.e.,

$$Y_L = \frac{W}{2} (1 - \cos(\phi_L - \beta))$$

In Case I  $\beta$  is positive so the target cross section is reduced while in Case III  $\beta$  is negative so it is expanded. Now, the detection angle for the  $i$ th piece of the target density projected onto the detection circle is

$$\phi_i = \cos^{-1} \left[ 1 - \frac{(1 - \cos(\phi_L - \beta))}{N} (i - 1/2) \right] \quad i = 1, 2, \dots, N$$

From here on, the intercept time and average value calculations are the same as before.

If  $\beta = 0$ , we have Case II, so we can use one general form for all three cases. This is what is done in PATROL.

The fraction of targets that are intercepted in each search leg are useful pieces of information. We will call them E and F for the parallel and diagonal legs.

$$E = \frac{2(\frac{w}{2} \sin \phi_L)}{\frac{w}{2} + \frac{w}{2}} = \sin \phi_L$$

$$F = \frac{\frac{w}{2}(1 - \cos(\phi_L - \beta))}{w} = \frac{1 - \cos(\phi_L - \beta)}{2}$$

The average intercept time  $T_I$  is calculated by weighing each search leg's average intercept time by the number of targets that will be intercepted. This is the product of the POD and the target cross section.

$$\bar{T}_I = E \cdot \text{POD}^P \bar{T}_I^P + F \cdot \text{POD}^D \bar{T}_I^D$$

This is the average intercept time for one type of vessel. To obtain the average intercept time for all types of traffic we need to consider the rate at which the traffic enters the region for the  $j$ th type vessel,  $r_j$ . The only type vessels that can be pursued are those that are slower than the cutter, i.e.,  $V > U_j$ . So here again we use the  $j \in$  notation. The final

form for the average intercept time is:

$$\bar{T}_I = \frac{\sum_{j \in \mathcal{E}} r_j [E_j \cdot \text{POD}_j^P \cdot \bar{T}_j^P + F_j \cdot \text{POD}_j^D \cdot \bar{T}_j^D]}{\sum_{j \in \mathcal{E}} r_j [E_j \cdot \text{POD}_j^P + F_j \cdot \text{POD}_j^D]}$$

Here the  $\bar{T}_j$ 's are the average intercept times on the parallel and diagonal legs for the  $j$ th type vessel.



## APPENDIX C

### EXAMPLE RUN

I N P U T S   F O R   C U T T E R  
R E C O R D   N U M B E R   860930001

CUTTER CLASS =        8

- 1 = Coast Guard Cutter
- 2 = Air Cushion Vehicle
- 3 = Catamaran
- 4 = Displacement
- 5 = Hybrid Vessel
- 6 = Hydrofoil, Fully Submerged
- 7 = Hydrofoil, Surface Piercing
- 8 = Planing
- 9 = Super Critical Planing
- 10 = Surface Effect Ship (SES)
- 11 = Small Waterplane Area Twin Hull (SWATH)
- 12 = Other

USER/DATE : Coast Guard R&D / 30 Sept 86  
DESCRIPTION: Notional Semi-Planing Patrol Boat

INPUTS FOR CUTTER  
RECORD NUMBER 860930001

DESIGN SPEED.....	32 KNOTS
DISPLACEMENT.....	200 TONS
LENGTH.....	155 FEET
BEAM.....	24 FEET
DRAFT.....	7 FEET
RANGE MAX SPEED.....	675 NMI
RANGE CRUISE SPEED.....	4400 NMI
ENDURANCE.....	10 DAYS
HORSEPOWER.....	4500 HP

MAIN ENGINE TYPE: Twin screws. Diesels, 2250 Hp each.  
COMMENT: Demonstration purposes only.

INPUTS FOR CUTTER  
RECORD NUMBER 860930001

MAXIMUM SPEED..... 32 KNOTS  
FUEL CAPACITY.....6750 GALS  
RADAR SWEEP WIDTH..... 30 NMI  
VISUAL SWEEP WIDTH..... 5 NMI  
RADAR AVAILABILITY..... 100 PERCENT  
MAX WAVE HEIGHT TO BOARD..... 8 FEET  
WAVE HGHT VS. MAX SPEED..... FEET, KNOTS

1, 32  
3, 28  
5, 22  
7, 15  
9, 12  
11, 8  
13, 7  
15, 5  
17, 0  
19, 0

SPEED VS. FUEL CONSM..... KNOTS, GPH

0, 11  
12, 50  
32, 285  
0, 0  
0, 0  
0, 0  
0, 0  
0, 0  
0, 0  
0, 0

INPUTS FOR SCENARIO  
RECORD NUMBER 860930001

SCREEN 1 OF 6

USER/DATE : Coast Guard R&D / 30 Sept 86  
DESCRIPTION: Cutter with WHEC as mother and WPB to take seized vessels.

A R E A

GEOGRAPHY

DISTANCE TO HOMEPORT..... 125 NMI  
DISTANCE TO REFUEL..... 30 NMI  
DISTANCE TO HANDOFF..... 50 NMI

ENVIRONMENT

AV 1/3 HIGHEST WAVES.....	FEET, PERCENT
0- 2, 20	
2- 4, 25	
4- 6, 15	
6- 8, 10	
8-10, 9	
10-12, 8	
12-14, 7	
14-16, 6	
16-18, 0	
18-20, 0	

INPUTS FOR SCENARIO  
RECORD NUMBER 860930001

SCREEN 2 OF 6

T R A F F I C

TYPE	MERCHANT	FISHING	PLEASURE	FAST
TOTAL VESSELS ENTERING PER WEEK: 400				
TRAFFIC PER WEEK (PERCENT)				
U.S. Special Interest	1	2	3	4
U.S. Other	2	7	10	5
Foreign Special Interest	2	6	6	5
Foreign Other	5	18	22	2
TRANSIT SPEED (KNOTS)				
All Registries/Interest	16	9	8	20
EVASION SPEED (KNOTS)				
All Registries/Interest	19	12	14	24

INPUTS FOR SCENARIO  
RECORD NUMBER 860930001

SCREEN 3 OF 6

TRAFFIC (CONT)

TYPE	MERCHANT	FISHING	PLEASURE	FAST
SMUGGLERS (PERCENT)				
U.S. Special Interest	12	15	18	24
U.S. Other	6	8	9	12
Foreign Special Interest	18	22	28	50
Foreign Other	9	11	14	25
CONTRABAND (TONS)				
U.S. Special Interest	10	2	1	2
U.S. Other	10	2	1	2
Foreign Special Interest	10	2	1	2
Foreign Other	10	2	1	2



INPUTS FOR SCENARIO  
RECORD NUMBER 860930001

## CHOICES

PATROL LENGTH..... 10 DAYS  
TRANSIT SPEED..... 16 KNOTS  
SEARCH AREA DIMENSIONS....130/ 75 NMI/NMI  
BARRIER LENGTH..... 130 NMI  
SEARCH SPEED..... 28 KNOTS  
SEARCH TACTIC..... 2  
\*1=Barrier Linear 2=Barrier Bowtie  
INTERCEPT SPEED..... 32 KNOTS  
MAX INTERCEPT TIME..... 1.5 HRS

I N P U T S   F O R   S C E N A R I O  
R E C O R D   N U M B E R   860930001

C H O I C E S   ( C O N T )

TYPE	MERCHANT	FISHING	PLEASURE	FAST
DETECTED PURSUED (PERCENT)				
All Flags	80	90	90	100
CLOSED ALONGSIDE (PERCENT)*				
U.S. Flag	60	95	95	100
Foreign Flag	75	98	98	100
*INDICATES CURRENTLY NOT USED BY MODEL				
BOARDED (PERCENT)				
U.S. Special Interest	100	100	100	100
U.S. Other	80	95	95	100
Foreign Special Interest	100	100	100	100
Foreign Other	90	95	95	100

INPUTS FOR SCENARIO  
RECORD NUMBER 860930001

## CHOICES (CONT)

TIME TO BOARD.....	0.5 HRS
TIME TO INSPECT.....	1.5 HRS
TIME TO SEIZE.....	1.5 HRS
TIME TO PROCESS VIOLATER....	2.5 HRS
TIME FOR SEARCH SNO.....	2.0 HRS
TIME FOR SEIZE SNO.....	2.0 HRS
INITIAL FUEL LEVEL.....	95 PERCENT
REFUEL WHEN DOWN TO.....	35 PERCENT
TIME TO REFUEL.....	2.5 HRS

. P A T R O L     R E S U L T S  
V E R S I O N   1 - O C T 8 6

PAGE 1

CUTTER

Coast Guard R&D / 30 Sept 86

860930001

Notional Semi-Planing Patrol Boat

SCENARIO

860930001

Coast Guard R&D / 30 Sept 86

Cutter with WHEC as mother and WPB to take seized vessels.

A L L O C A T I O N     O F     E F F O R T

PHASE	DISTANCE (N.M.)	TIME (HRS)	PROBABILITY (TIME)
ENTIRE PATROL	1951	239	1.00
IN OPERATING CYCLE	1449	192	0.80
Search	640	32	0.13
Intercept	398	19	0.08
Inspect	0	88	0.37
Escort	411	53	0.22
NOT IN OPERATING CYCLE	502	47	0.20
Transit	249	18	0.08
Refuel	253	29	0.12

ESCORT = ESCORT + PROCESS + TRANSIT BACK

REFUEL = TRANSIT + REPLENISH + TRANSIT BACK

T R A F F I C     S U M M A R Y

SMUGGLERS	(#)
TOTAL	91
U.S.	23
Foreign	68
CONTRABAND	(TONS)
SHIPPED	195
Missed	188
Intercepted	7

P A T R O L      R E S U L T S  
V E R S I O N    1 - O C T 8 6

PAGE 2

CUTTER

Coast Guard R&D / 30 Sept 86  
Notional Semi-Planing Patrol Boat  
SCENARIO

860930001

Coast Guard R&D / 30 Sept 86

860930001

Cutter with WHEC as mother and WPB to take seized vessels.

V E S S E L    P E R F O R M A N C E

\*EVENT TRACE

O C C U R R E N C E S  
( # )

Detections			45
Classified No Interest		6	
Intercepted		39	
Not Boarded		13	
Boarded/Inspected		26	
U.S. Flag	9		
For. Flag	17		
Not Violators		22	
Violators Seized		4	
U.S. Flag	1		
For. Flag	3		

AVERAGE TIME

HRS

Between Detections	0.71
To Intercept	0.49
To Board & Inspect	3.35
To Escort, Handoff & Return	12.78

CUTTER SEARCHING CAPABILITY

POD

In Barrier (Pattern Search)	0.59
While Intercepting (Random)	0.44
While Standing-By Boarding (0 kts)	0.23
Overall On-Scene POD	0.34

MISSION MEASURES OF EFFECTIVENESS  
(For Entire Patrol 239 Hrs)

RATE  
(PERCENT)

Fraction of all Vessels Detected	21.4
Fraction of all Vessels Intercepted	6.9
Fraction of all Vessels Boarded	4.6
Fraction of all Smugglers Seized	4.5
Fraction of all Tonnage Seized	3.6

\*Events accurate to within 1 occurrence

P A T R O L     R E S U L T S  
V E R S I O N   1 - O C T 8 6

## CUTTER

860930001

Coast Guard R&amp;D / 30 Sept 86

Notional Semi-Planing Patrol Boat

## SCENARIO

860930001

Coast Guard R&amp;D / 30 Sept 86

Cutter with WHEC as mother and WPB to take seized vessels.

L O G I S T I C S     I N F O R M A T I O N

M I S S I O N   P R O F I L E S

ACTIVITY	SPEED (KTS)	TIME ( P E R C E N T )	DISTANCE	FUEL
Standby	0.0	45.6	0.0	5.8
Escort	8.4	11.3	10.5	6.0
Transit	13.6	21.6	36.2	24.4
Search	19.9	13.4	32.8	37.1
Intercept	20.7	8.0	20.4	26.6
TOTALS		239 HRS	1951 N.M.	20625 GALS

D E T A I L E D   B R E A K O U T

OPERATION	SPEED (KTS)	TIME (HRS)	DISTANCE (NM)	FUEL (GALS)
Transit to and from Homeport	13.6	18.4	250	1781
Transit to and from Refuel Pt.	13.6	18.5	251	1795
Replenish with Fuel	0.0	10.5	0	115
Escort Violators to Hand-off Pt.	8.4	27.1	205	1237
Process Violator in System	0.0	10.3	0	112
Transit Back to Op-Area	13.6	15.1	205	1461

4 REFUELING EVOLUTIONS, TAKING 19324 GALLONS ON BOARD.

41 % OF PATROL SPENT IN LOGISTICS TASKS.

P R E M A R K O V C A L C U L A T I O N S  
V E R S I O N 1 - O C T 8 6

## CUTTER

860930001

Coast Guard R&amp;D / 30 Sept 86

Notional Semi-Planing Patrol Boat

860930001

## SCENARIO

Coast Guard R&amp;D / 30 Sept 86

Cutter with WHEC as mother and WPB to take seized vessels.

CUTTER SPEEDS	INPUT SPEED	DEGRADED SPEED	FUEL COMSUMPTION
Standby Speed	0.00	0.00	11.0
Escort Speed	10.66	8.40	45.7
Transit Speed	16.00	13.61	97.0
Search Speed	28.00	19.91	238.0
Intercept Speed	32.00	20.71	285.0

## TRAFFIC SPEEDS

MASK

Merchant Transit	16.00	14.91	
Fish Transit	9.00	7.98	
Pleasure Transit	8.00	6.20	
Fast Transit	20.00	14.31	
Merchant Evasion	19.00	17.71	1.0
Fish Evasion	12.00	10.63	1.0
Pleasure Evasion	14.00	10.85	1.0
Fast Evasion	24.00	17.17	1.0

## ARRIVAL RATE FOR EACH CLASS OF VESSELS PER HOUR (rj) :

Merchant	Fishing	Pleasure	Fast
0.24	0.79	0.98	0.38

OVERALL VESSEL ARRIVAL RATE PER HOUR (RDOT) : 2.38

## D E T E C T I O N

GAMMA (KTS)	U (KTS)	R (#/HR)	P O D (#)	ND (#/HR)	TD (HRS)
1.34	14.9	0.2380952	0.362	0.086	11.606
2.50	8.0	0.7857143	0.590	0.463	2.159
3.21	6.2	0.9761903	0.739	0.721	1.387
1.39	14.3	0.3809523	0.372	0.142	7.058

TOTAL DETECTIONS PER HOUR = 1.4123257

AVERAGE TIME BETWEEN DETECTIONS = 0.7080520 HOURS



P R E M A R K O V C A L C U L A T I O N S  
( C O N T I N U E D )  
V E R S I O N 1 - O C T 8 6

CUTTER 860930001  
Coast Guard R&D / 30 Sept 86  
Notional Semi-Planing Patrol Boat  
SCENARIO 860930001  
Coast Guard R&D / 30 Sept 86  
Cutter with WHEC as mother and WPB to take seized vessels.

I N T E R C E P T

U (KTS)	INTERCEPT TIMES		ROTATION ANGLE	SWEEP COVERAGE		WEIGHTED POD
	PARALLEL	DIAGONAL		PARALLEL	DIAGONAL	
17.7	0.44	0.59	16.70	1.0000	0.5658	0.282
10.6	0.44	0.66	-6.59	1.0000	0.9967	0.588
10.9	0.35	0.49	-12.57	1.0000	0.9880	0.734
17.2	0.44	0.63	14.62	1.0000	0.6396	0.304

AVERAGE INTERCEPT TIME = 0.4859605 HRS  
FRACTION OF TARGETS WITHIN MAX INT TIME = 0.9639888  
FRACTION OF TARGETS NOT WITHIN MAX INT TIME = 0.0360112

DETECTION RATE (DDOT) = 1.41233

INTERCEPTION RATE (IDOT) = 1.23017

BOARDING RATE (BDOT) = 1.18441

VESSEL SEIZURE RATE (SVDOT) = 0.18344

TONNAGE SEIZURE RATE (TSZDOT) = 0.31471

SMUGGLER ARRIVAL RATE (SMGDOT) = 0.38024

TONNAGE ARRIVAL RATE (TNDOT) = 0.81286

FRACTION BOARDED U.S. (BUS) = 0.32475

FRACTION BOARDED FOREIGN (BFOR) = 0.67525

FRACTION SEIZED U.S. (SUS) = 0.24456

FRACTION SEIZED FOREIGN (SFOR) = 0.75544

FRACTION OF CASES THAT CAN BE BOARDED (K) = 0.05000

FRACTION OF CASES THAT CAN NOT BE BOARDED (1-K) = 0.95000

P R E M A R K O V C A L C U L A T I O N S  
 ( C O N T I N U E D )  
 V E R S I O N 1 - O C T 8 6

CUTTER 860930001  
 Coast Guard R&D / 30 Sept 86  
 Notional Semi-Planing Patrol Boat  
 SCENARIO 860930001  
 Coast Guard R&D / 30 Sept 86  
 Cutter with WHEC as mother and WPB to take seized vessels.

PROBABILITY TRANSITION MATRIX (P)

.13	.87	0.0	0.0
.33	0.0	.67	0.0
.85	0.0	0.0	.15
1.0	0.0	0.0	0.0

PROBABILITY OF ESCORTING JTH VESSEL (PEE(J)):  
 PEE(1) = 0.02874 PEE(2) = 0.27725 PEE(3) = 0.51433 PEE(4) = 0.17968

M A R K O V C A L C U L A T I O N S  
V E R S I O N 1 - O C T 8 6

## CUTTER

860930001

Coast Guard R&amp;D / 30 Sept 86

Notional Semi-Planing Patrol Boat

## SCENARIO

860930001

Coast Guard R&amp;D / 30 Sept 86

Cutter with WHEC as mother and WPB to take seized vessels.

## EMBEDDED CHAIN PROBABILITIES :

PI(1) = 0.39

PI(2) = 0.34

PI(3) = 0.23

PI(4) = 0.04

WEIGHTED TIME (WEIGT) = 1.67

## PHASE PROBABILITIES :

P(1) = 0.17

P(2) = 0.10

P(3) = 0.46

P(4) = 0.27

P O S T M A R K O V C A L C U L A T I O N S  
V E R S I O N 1 - O C T 8 6

CUTTER 860930001

Coast Guard R&D / 30 Sept 86  
Notional Semi-Planing Patrol Boat

SCENARIO 860930001

Coast Guard R&amp;D / 30 Sept 86

Cutter with WHEC as mother and WPB to take seized vessels.

ONE WAY TRANSIT FOR REFUEL (T1REF) = 2.2

ONE WAY TRANSIT FOR TRANSIT (T1TRA) = 9.2

SINGLE REFUELING CYCLE TIME (TREFCY) = 6.9

TOTAL TIME IN TRANSIT PHASE (TOTTRA) = 18.4

ESCORT SEIZED VESSEL TO HANDOFF (T1ESC) = 6.6

TRANSIT RETURN FROM HANDOFF (TTRESC) = 3.7

SINGLE ESCORT CYCLE TIME (TESCCY) = 12.8

JTH VESSEL VELOCITY IN ESCORT MOVING (VELESC(J)) :

VELESC(1) = 14.23

VELESC(2) = 10.06

VELESC(3) = 8.52

VELESC(4) = 13.95

PROBABILITY OF ESCORTING JTH VESSEL (PEE(J)) :

PEE(1) = 0.03

PEE(2) = 0.28

PEE(3) = 0.51

PEE(4) = 0.18

AVERAGE TWO WAY ESCORT VELOCITY (VBARESC2) = 10.1

AVERAGE FUEL CONSUMPTION (GDOT) = 85.7

FUEL CAPACITY (FC) = 6750

INITIAL FUEL LEVEL (FI) = 0.95

REFUEL LEVEL (FR) = 0.35

FUEL INITIAL (FUELINIT) = 6412.5

FUEL LOW (FUELOW) = 2362.5

FUEL ON-SCENE INITIALLY (FULOSI) = 5521.6

FUEL ON-SCENE OTHER (FULOSO) = 6536.2

DELTA FUEL INITIAL (DELFULI) = 3159.1

P O S T M A R K O V C A L C U L A T I O N S  
( C O N T I N U E D )  
V E R S I O N 1 - O C T 8 6

CUTTER 860930001  
Coast Guard R&D / 30 Sept 86  
Notional Semi-Planing Patrol Boat  
SCENARIO 860930001  
Coast Guard R&D / 30 Sept 86  
Cutter with WHEC as mother and WPB to take seized vessels.

DELTA FUEL OTHER (DELFULO) = 4173.7  
TIME ON-SCENE INITIAL (TOSI) = 36.8  
TIME ON-SCENE OTHER (TOSO) = 48.7  
TIME IN AREA (TINAREA) = 221.6  
NUMBER OTHER CYCLE (NUMOCY) = 3.2  
TOTAL TIME ON-SCENE (TOST) = 192.6  
NUMBER REFUELINGS (NUMREF) = 4.7  
TIME IN SEARCH PHASE (TS) = 32.2  
TIME IN INTERCEPT PHASE (TI) = 19.2  
TIME IN BOARD PHASE (TB) = 88.7  
TIME IN ESCORT PHASE (TE) = 53.4  
NUMBER OF DETECTIONS (ND) = 45.4  
NUMBER OF INTERCEPTIONS (NI) = 39.6  
NUMBER OF BOARDINGS (NB) = 26.5  
NUMBER OF SEIZURES (NZ) = 4.6  
NUMBER NO INTERCEPTIONS (NNI) = 6.9  
NUMBER NO BOARDINGS (NNB) = 13.1  
NUMBER NO VIOLATERS (NNV) = 22.4  
NUMBER U.S. BOARDED (NUSB) = 9.6  
NUMBER FOR. BOARDED (NFORB) = 17.9  
NUMBER U.S. SEIZED (NUSZ) = 1.5  
NUMBER FOR. SEIZED (NFORZ) = 3.6  
DISTANCE IN SEARCH PHASE (DS) = 640.7

P O S T M A R K O V C A L C U L A T I O N S  
 ( C O N T I N U E D )  
 V E R S I O N 1 - O C T 8 6

CUTTER	860930001
Coast Guard R&D / 30 Sept 86	
Notional Semi-Planing Patrol Boat	
SCENARIO	860930001
Coast Guard R&D / 30 Sept 86	
Cutter with WHEC as mother and WPB to take seized vessels.	
DISTANCE IN INTERCEPT PHASE (DI) =	398.6
DISTANCE IN BOARD PHASE (DB) =	0.0
DISTANCE IN ESCORT PHASE (DE) =	411.3
DISTANCE ON-SCENE TOTAL (DOST) =	1449.5
NUMBER REFUELINGS (NREF) =	4.7
TOTAL TIME TRANSIT FOR REFUELING (TOTRREF) =	18.5
TOTAL TIME REPLENISHING (TOTREP) =	10.5
TOTAL TIME IN REFUELING PHASE (TOTREF) =	29.0
DISTANCE STEAMED FOR REFUELING (DITREF) =	254.0
DISTANCE STEAMED TRANSITING (DITTRA) =	250.0
TOTAL TIME ESCORTING TO HANDOFF PT. (TOTESC) =	27.1
TOTAL TIME TRANSITING BACK TO OP. AREA (TOTRAESC) =	15.1
TOTAL TIME PROCESSING SEIZED VES. (TOTPRO) =	10.3
TOTAL TIME IN STANDBY OP. (TOSTBOP) =	109.5
TOTAL TIME IN ESCORT OP. (TOESOP) =	27.1
TOTAL TIME IN TRANSIT OP. (TOTRAOP) =	52.0
TOTAL TIME IN SEARCH OP. (TOSCHOP) =	32.2
TOTAL TIME IN INTERCEPT OP. (TOINTOP) =	19.2
TOTAL PATROL TIME (TOTPAT) =	240.0
TOTAL TIME IN LOGISTICS TASKS (TOTILOG) =	99.8
PERCENT TIME IN LOGISTICS TASKS (PCTTLOG) =	0.42
FUEL IN STANDBY OP. (FSTBOP) =	1204.6
FUEL IN ESCORT OP. (FESOP) =	1238.0

P O S T M A R K O V C A L C U L A T I O N S  
( C O N T I N U E D )  
V E R S I O N 1 - O C T 8 6

CUTTER 860930001  
Coast Guard R&D / 30 Sept 86  
Notional Semi-Planing Patrol Boat  
SCENARIO 860930001  
Coast Guard R&D / 30 Sept 86  
Cutter with WHEC as mother and WPB to take seized vessels.

FUEL IN TRANSIT OP. (FTRAOP) = 5039.7  
FUEL IN SEARCH OP. (FSCHOP) = 7658.5  
FUEL IN INTERCEPT OP. (FINTOP) = 5485.0  
TOTAL FUEL ON PATROL (FUTOPAT) = 20625.7  
DISTANCE TRAVELED IN STANDBY (DSTB) = 0.0  
DISTANCE TRAVELED IN ESCORT (DESC) = 205.1  
DISTANCE TRAVELED IN TRANSIT (DTRA) = 707.1  
DISTANCE TRAVELED IN SEARCH (DSCH) = 640.7  
DISTANCE TRAVELED IN INTERCEPT (DINT) = 398.6  
TOTAL DISTANCE TRAVELED ON PATROL (DITOPAT) = 1951.5

P R O F I L E S

T1= 0.46	F1= 0.06	D1= 0.00
T2= 0.11	F2= 0.06	D2= 0.11
T3= 0.22	F3= 0.24	D3= 0.36
T4= 0.13	F4= 0.37	D4= 0.33
T5= 0.08	F5= 0.27	D5= 0.20

FUEL CONSUMED ON TRANSIT TO REPLENISH (FCONTR) = 213.8  
FUEL TAKEN ONBOARD (FTOB) = 4601.3  
TOTAL FUEL TAKEN ONBOARD (FTTOB) = 19324.6  
DISTANCE TRANSIT TO AND FROM HOMEPORT (DTOTRA) = 250.0  
DISTANCE TRANSIT REFUELING (DTOREF) = 252.0  
DISTANCE REPLENISH (DTOREP) = 0.0  
DISTANCE ESCORT (DTOESC) = 205.1  
DISTANCE PROCESSING (DTOPRO) = 0.0  
DISTANCE TRANSIT BACK FROM ESCORT (DTOTRAESC) = 205.1



P O S T M A R K O V C A L C U L A T I O N S  
( C O N T I N U E D )  
V E R S I O N 1 - O C T 8 6

CUTTER 860930001  
Coast Guard R&D / 30 Sept 86  
Notional Semi-Planing Patrol Boat

SCENARIO 860930001  
Coast Guard R&D / 30 Sept 86  
Cutter with WHEC as mother and WPB to take seized vessels.  
FUEL TRANS HOMEPORT (FTOTRA) = 1781.8

FUEL TRANS REFUELING (FTOREF) = 1795.9

FUEL REPLENISH (FTOREP) = 115.5

FUEL ESCORT (FTOESC) = 1238.0

FUEL PROCESS (FTOPRO) = 112.8

FUEL TRANS ESC (FTOTRAESC) = 1462.0

DISTANCE OUT MARKOV (DNOST) = 503.0

TIME OUT MARKOV (TNOST) = 47.4

PROBABILITIES IN ALLOCATION:

P1 = 0.13

P2 = 0.08

P3 = 0.37

P4 = 0.22

P5 = 0.08

P6 = 0.12

P7 = 0.80

P8 = 0.20

P9 = 1.00

TOTAL VESSELS ENTIRE PATROL (TOTVES) = 571.4

DETECTION RATE IN SEARCH (DD) = 0.08

INTERCEPTION RATE (II) = 0.07

BOARDING RATE (BB) = 0.05

TOTAL SMUGGLER ENTIRE PATROL (TOTSMG) = 91.3

VESSEL SEIZURE RATE (ZZ) = 0.04

TOTAL CONTRABAND ENTIRE PATROL (TOTCON) = 195.1

AVERAGE TONS PER SEIZURE (TONBAR) = 1.7

TOTAL TONNAGE INTERDICTED (TONINT) = 7.0

INTERDICTION RATE (IN) = 0.04

U229530